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Again, where the central line meets the west coast of Africa, in about 13° 20′ east longitude, and 9° 42′ south latitude. At this point

	h	m	s	
Totality begins Dec. 22 at	2	56	3	local M.T.
Duration of totality		3	12	

Sun's altitude, 46°.

For any point near this the Greenwich mean time of beginning and ending of totality may be found from the formulæ,

$$\cos\omega = +8{\cdot}9188 - [1{\cdot}93737]\sin l + [1{\cdot}57729]\cos l \,.\, \cos(\lambda + 217^\circ\ 36'{\cdot}6)$$

$$t = 0^{\mathrm{h}}\ 19^{\mathrm{m}}\ 45^{\mathrm{s}}{\cdot}5 \mp [1{\cdot}98325]\sin\omega + [2{\cdot}59376]\sin l$$
$$- [3{\cdot}95966]\cos l \,.\, \cos(\lambda + 121^\circ\ 32'{\cdot}4),$$

where l is the *geocentric* latitude, λ the longitude from Greenwich taken *positively*, and the quantities within square brackets are the logarithms of seconds of time.

The central line on entering Africa runs through the following points:

Greenwich Mean Time.		Longitude E.		Latitude S.	
h	m	°	′	°	′
2	4	13	10·3	9	45·1
2	5	13	41·3	9	34·8
2	6	14	12·7	9	24·2
2	7	14	44·5	9	13·4
2	8	15	16·9	9	2·2

Remarks on Sir George Airy's "Numerical Lunar Theory."

By Professor J. C. Adams, M.A., &c.

In the Report of the Council on the subject of Sir George Airy's "Numerical Lunar Theory," it has been explained that the large discordances which have been found by the author to result from the substitution of the values of the Moon's coordinates, as found by Delaunay, in the differential equations of motion, are caused by the large errors of Delaunay's coefficients of parallax, which Sir George has employed. It may be useful and not uninteresting to give on this subject some additional details. In the first place it will be well to prevent a possible misapprehension. In speaking of the errors of Delaunay's coefficients it is not intended to imply that there is any mistake in Delaunay's *theory*. The terms of the analytical expression for the Moon's parallax which Delaunay gives are all correct, but they only extend to the fifth order of small quantities, and are therefore not nearly precise enough to be used for the purpose to which the expression for the parallax is applied by Sir George

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Airy. Delaunay intended this value of the parallax to be employed merely in reducing the apparent place of the Moon to its place as seen from the Earth's centre, and for this purpose the value is perhaps sufficiently accurate.

If the several transformations of the elements given by Delaunay in his great work had been applied to the analytical expression for the reciprocal of the radius vector, and if Delaunay had carried the developments to the same extent as he had done in the case of the Moon's longitude and latitude, the theory would have been quite competent to give the third coordinate with the same degree of precision as had been attained in the case of the two other coordinates.

The following table, which is reduced from the table given in pp. 398, 399, of vol. xliii. of the *Monthly Notices*, R.A.S., shows the proportional values of the coefficients of parallax as found by me, mainly after Pontécoulant, when compared with those employed by Sir George Airy after Delaunay.

Argt.	My Coefficient.	Delaunay's.
o	10000000,	10000000,
l	544989, 3	545145, 6
$2D-l$	100236, 0	99822, 4
$2D$	82493, 6	82329, 2
$2l$	29716, 6	29796, 4
$2D+l$	9029, 0	8950, 8
$2D-S$	5595, 6	5482, 2
$2D-l-S$	4234, 0	4243, 1
$l-S$	3380, 7	3074, 5
D	− 2773, 0	− 2739, 9
$l+S$	− 2770, 0	− 2664, 0
$2f-l$	− 2074, 6	− 2068, 25
$3l$	1835, 0	1844, 7
$4D-l$	1753, 2	1458, 2
S	− 1168, 8	− 1248, 4
$2D-l+S$	− 675, 0	− 1107, 0
$2D+S$	− 894, 1	− 957, 1
$4D-2l$	1087, 0	906, 9
$2D-2l$	− 897, 05	− 809, 3
$2D+2l$	821, 0	790, 9
$2D+l-S$	648, 7	574, 7
$4D$	759, 7	572, 6
$D+S$	423, 7	440, 3
$2D-2f$	− 309, 7	− 319, 0
$2l-S$	359, 4	300, 9

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Argt.	My Coefficient.	Delaunay's.
$2D-3l$	− 338,95	− 295,7
$D+l$	− 309,7	− 283,7
$2l+S$	− 292,2	− 267,9
$2D-2f-l$	− 251,3	− 238,4
$2D-2S$	260,05	222,0
$2D-2f+l$	− 140,25	− 146,4
$2D-l-2S$	146,1	132,6
$4l$	119,8	121,0
$2D+l+S$	− 137,3	− 117,2
$4D-l-S$	184,1	86,8
$3D-l$	− 105,2	− 58,1
$4D+l$	93,5	54,05
$2D+3l$	51,2	51,
$4D-2l-S$	87,7	46,5
$3D$	14,6	46,2
$2D+2f-2l$	− 42,6	− 43,
$D+l+S$	38,9	38,9
$l-2S$	38,3	38,3
$2D-l+2S$	− 37,4	− 37,4
$2D-2l-S$	− 37,1	− 37,1

This table shows at a glance how great the errors of Delaunay's coefficients of parallax, when reduced to the form in which they are employed by Sir George Airy, in many cases really are. Hence the discordances which he met with in the results of the substitutions should occasion no surprise. In the Introduction to the "Numerical Lunar Theory," p 4, line 20, it is stated through inadvertence that the factor which Sir George Airy calls M is a quantity "depending on the proportion of the masses of the Earth and Moon." This is not the case however, since M is simply the ratio of the sum of the actual masses of the Earth and Moon to the sum of the masses which would be required to make the Moon describe an undisturbed orbit about the Earth in which the periodic time and the mean parallax were the same as in the actual orbit.

The theoretical value of M is simply expressed as the *cube* of the constant term in Delaunay's value of $\frac{a}{r}$. This value is given analytically in p. 802 or p. 914 of the second volume of Delaunay's Theory, but only to the fifth order of small quantities, which is not accurate enough. The development of the constant term of $\frac{a}{r}$ has been carried by me to a much greater extent at p. 472 of vol. xxxviii. of the *Monthly Notices*. Turning this expres-

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sion into numbers, and cubing it, we find the value of M to be 1·0027259, which agrees very closely with the value found by Sir George Airy by comparing the constant terms on the two sides of his equation (10).

The other two ways of finding M proposed by Sir George in p. 76 of his Theory, viz. by comparing the quantities on the two sides of the equations (10) and (12), corresponding to the arguments 2 and 301 respectively, are not satisfactory, as the results will be affected by errors in the theoretical determinations of the mean motions of the Moon's perigee and node respectively.

The multiplier M, representing the sum of the masses of the Earth and Moon, must be employed wherever the mutual attraction of these two bodies comes in question. In Sir George Airy's note at p. 254 of the March number of the *Monthly Notices*, he calls M the coefficient of a *solar* term, but this is plainly a mistake. I should mention that I have already communicated the substance of this paper to Sir George Airy himself.

The Positions for 1750·0 *and Proper Motions of* 154 *Stars south of* −29° *declination deduced from a revision of Powalky's Reduction of the Star Places of Lacaille's Astronomiæ Fundamenta.*

By A. M. W. Downing, M.A.

In the report of the Superintendent of the U.S. Coast and Geodetic Survey for the year ending June 1882, the late Dr. C. R. Powalky published "a new reduction of Lacaille's observations, made at the Cape of Good Hope and at Paris between 1749 and 1757, and given in his *Astronomiæ Fundamenta*, together with a comparison of the results with the Bradley-Bessel *Fundamenta*; also a catalogue of the places of 150 stars south of declination −30°, for the epochs 1750 and 1830." Powalky's catalogue thus depends on the star places of Bessel's *Fundamenta*, that is to say, the assumed right ascensions of the stars used for determination of clock error are those of the *Tabulæ Regiomontanæ* or of the *Fundamenta*, and systematic corrections to Lacaille's declinations have been determined by comparing them with the same authorities. It is the object of the present paper to deduce the necessary corrections to Powalky's places of the stars observed at the Cape, so as to make them depend on Auwers' re-reduction of Bradley, and to determine the proper motions resulting from a comparison of these corrected places with the mean for the epoch 1875 of Stone's Cape Catalogue, and Gould's Argentine General Catalogue. The various processes of reduction are not very clearly explained by Powalky, and on my writing to Professor Peters, of Clinton (who examined and reported on the paper), to ask if he could supply me with any further information, he had the great kindness to cause a search